The only way to learn mathematics is to do mathematics.

PAUL HALMOS

Exponential and Logarithmic Functions - Test Yourself

Introduction

Negative exponents:	$a^{-n} = \frac{1}{a^n}, \frac{1}{a^{-n}} = a^n, a \neq 0$
Product Rule:	$a^m \cdot a^n = a^{m+n}$
Quotient Rule:	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
Power Rule:	$(a^m)^n = a^{mn}$
Raising a product to a power:	$(ab)^n = a^n b^n$
Raising a quotient to a power:	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0;$
	$\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}, b \neq 0, a \neq 0$
Scientific notation:	$M \times 10^n$, or 10^n, where $1 \leq M < 10$

1. $21ab^3c^4 \cdot 3a^3c$ equals:

- $63a^3b^3c^4$
- $63a^4b^4c^5$
- $63a^4b^3c^5$
- $63a^3b^4c^5$

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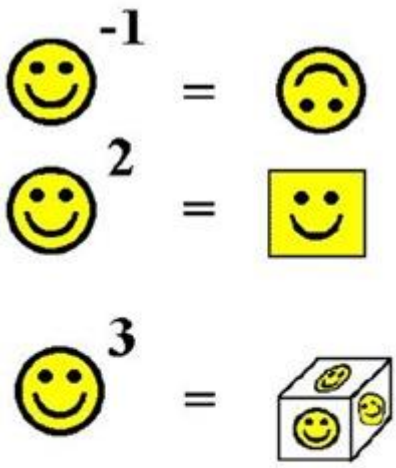
2. $\left(\frac{3d^2e^2}{d^3ef^2}\right)^3$ equals:

- $\frac{27e^3}{d^3f^6}$
- $\frac{9e^3}{d^3f^6}$
- $\frac{27e^6}{d^3f^6}$
- $\frac{27e^3}{d^3f^5}$

3. $6g^0 \cdot (7g^3)^0$ equals:

○ $42g^{30}$ ○ $42g^3$ ○ 42 ○ 6



4. $j^{-2}k \cdot j^{-4}k^{-1}$ equals:

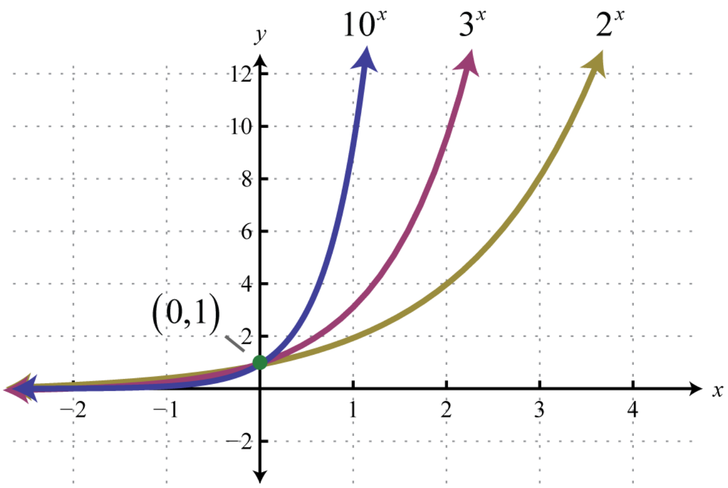
○ $\frac{k}{j^6}$ ○ $\frac{1}{j^6}$ ○ $\frac{1}{j^2}$ ○ $\frac{k}{j^2}$

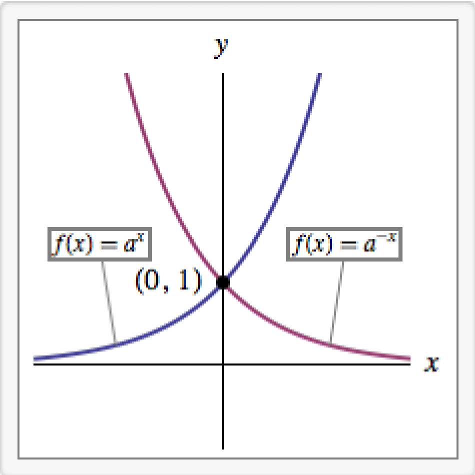
---

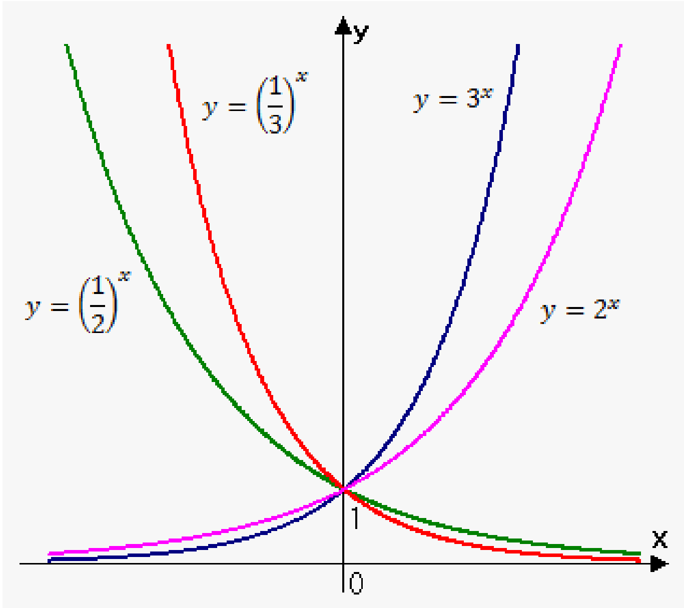
5. If $x = 2$, then $4x^2 \cdot \left(\frac{16}{x^3}\right)^2 : \frac{2x^3}{8 \cdot (4x)^{-2}}$ equals:

○ $\frac{1}{8}$ ○ $\frac{1}{2}$ ○ 4 ○ 8

Exponential Functions







6. All of the following are exponential functions except:

◯ $f(x) = \left(\frac{1}{3}\right)^x$ ◯ $f(x) = 1^x$ ◯ $f(x) = 2^x$ ◯ $f(x) = 4^x$

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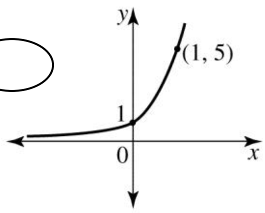
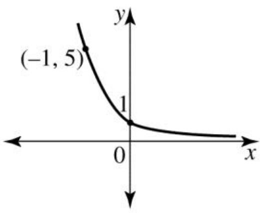
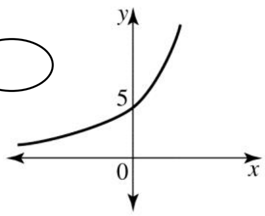
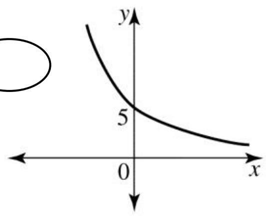
7. The graph of $y = 5^x$ is best represented by:

◯ 

◯ 

◯ 

◯ 

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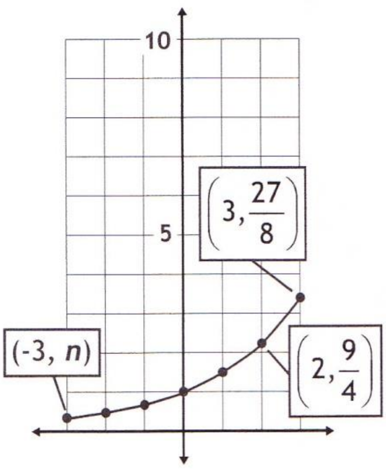
8. The point $(-3, n)$ exists on the exponential graph shown on the right.

The value of n is:

◯ $-\frac{8}{27}$ ◯ $\frac{1}{3}$

◯ $\frac{2}{3}$ ◯ $\frac{8}{27}$



9. The graph of $y = \left(\frac{1}{2}\right)^{x+3} - 2$ has:

- ◯ A vertical asymptote at $x = -3$
- ◯ A vertical asymptote at $y = -2$
- ◯ A horizontal asymptote at $x = -3$
- ◯ A horizontal asymptote at $y = -2$

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10. The function $y = 25 \cdot 5^{x}$ has the same graph as:

◯ $y = 5^{x+3}$ ◯ $y = 5^{x+2}$ ◯ $y = \left(\frac{1}{5}\right)^{2x}$ ◯ $y = \left(\frac{1}{5}\right)^{3x}$

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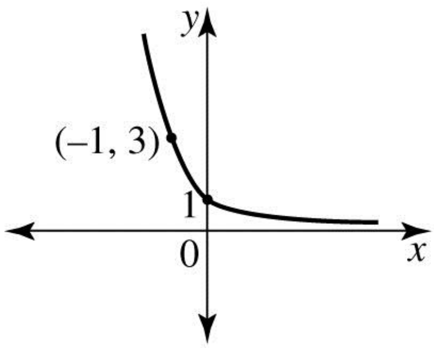
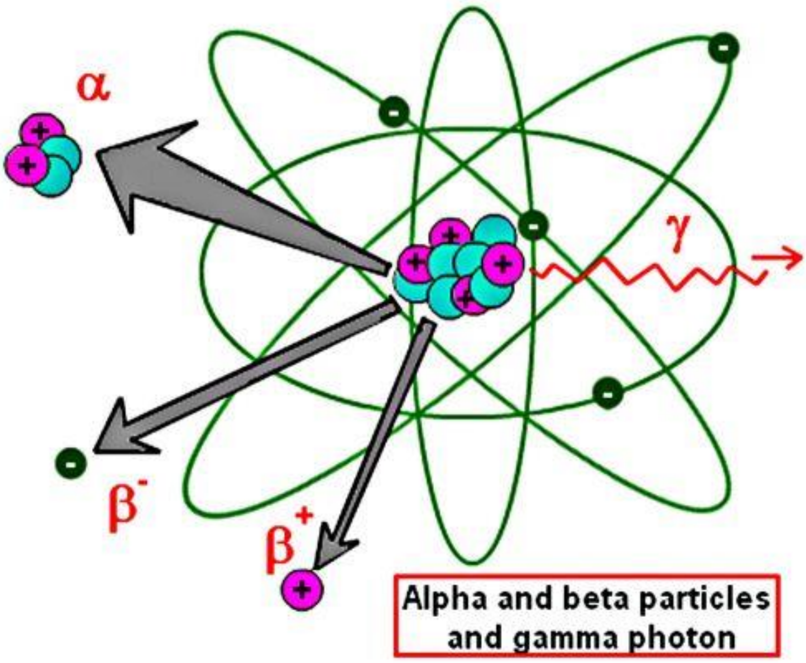
11. The figure below represents the graph with the equation:



◯ $y = 3^{-x}$ ◯ $y = -3x$ ◯ $y = 3 \cdot 2^{x}$ ◯ $y = 3 \cdot 2^{-x}$

Radioactive Decay and Half-life



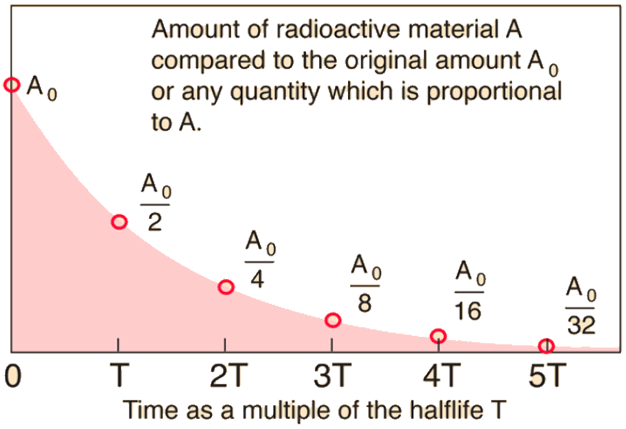
Alpha and beta particles and gamma photon

Each radioactive element, or radionuclide, has a special **half-life**.

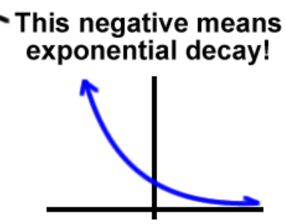
The **half-life** is the time taken for half of the atoms of a radioactive substance to decay.

Half-lives can range from less than a millionth of a second to millions of years depending on the element concerned.

After one **half-life** the level of radioactivity of a substance is halved, after two half-lives it is reduced to one quarter, after three half-lives to one-eighth, and so on.



$$A = A_0 \cdot 2^{-t/h}$$



12. The radioactive isotope **gallium 67 (^{67}Ga),** used in the diagnostic of malignant tumors, has a biological half-life of 46.5 hours. If we start with 100 milligrams of the isotope, how many milligrams will be left after :

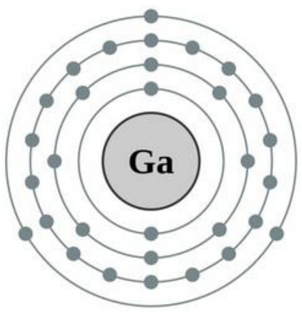
a) 24 hours ,

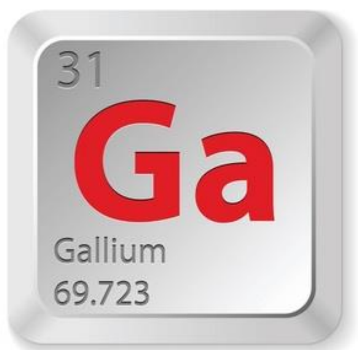
_____________ milligrams

b) 1 week ?

_____________ milligrams

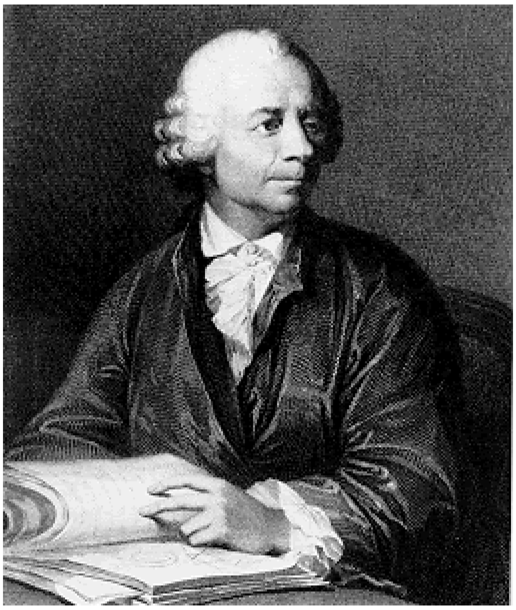
Compute answers to three significant places.

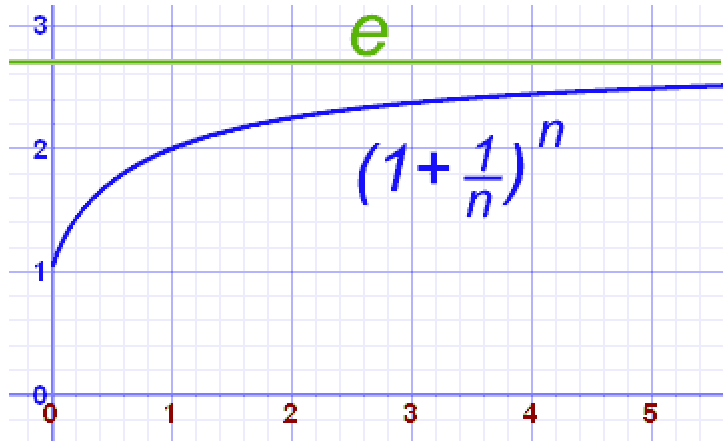




The Natural Exponential Function:

$$f(x) = e^x$$





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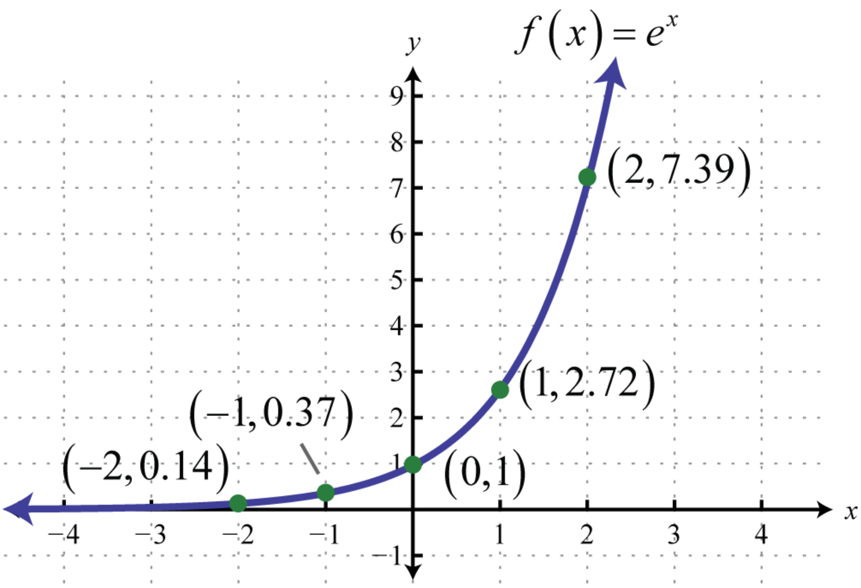
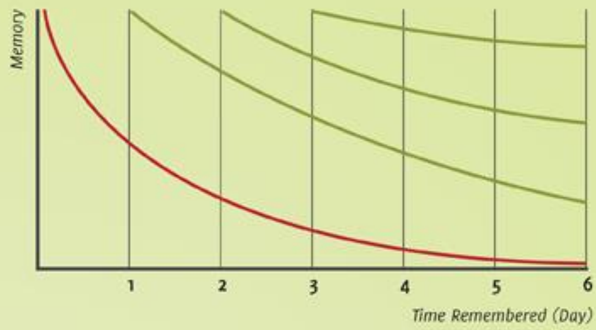


MEMORY RETENTION — *and the* — FORGETTING CURVE

Our brain houses many, many memories, but why do we remember some things so strongly and have a difficult time recalling others? Consider the forgetting curve:

THE EXPONENTIAL NATURE OF FORGETTING

In the 19th century, psychologist Hermann Ebbinghaus explored the exponential nature of forgetting. He came up with the following:



The following formula explains the curve:

$$R = e^{-\frac{t}{s}}$$

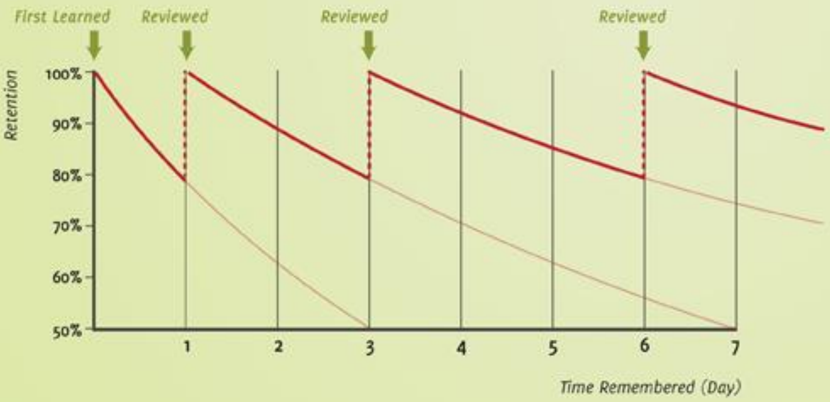
R = memory retention
S = strength of memory
T = time

The curves hypothesize the decline of memory retention over time.

Forgetting happens most rapidly right after learning occurs; it then slows as time passes.

REVIEWING TO REMEMBER

A typical forgetting curve shows that our newly learned knowledge and made memories are halved in a matter of days or weeks unless the information is reviewed.



After learning something, our memory of it will decline over time unless we review it. The more review it, the stronger we make the memory, the longer we can remember it.

When exposed to the same material repeatedly, it takes less time to pull the information from your long-term memory.

How quickly we forget things depends on a number of factors, including:

The difficulty of the material

How meaningful the material is to us

How the material was learned

If the material was frequently learned or remembered

Physiological factors like stress and sleep

Other memories, called flashbulb memories, are so vividly imprinted in our minds that we remember them easily, like the 9/11 attacks.

Sources: indiana.edu | ellaz.com | psychology.about.com | sidsavara.com | en.wikipedia.org

Information provided by:
http://www.onlinecolleges.net/

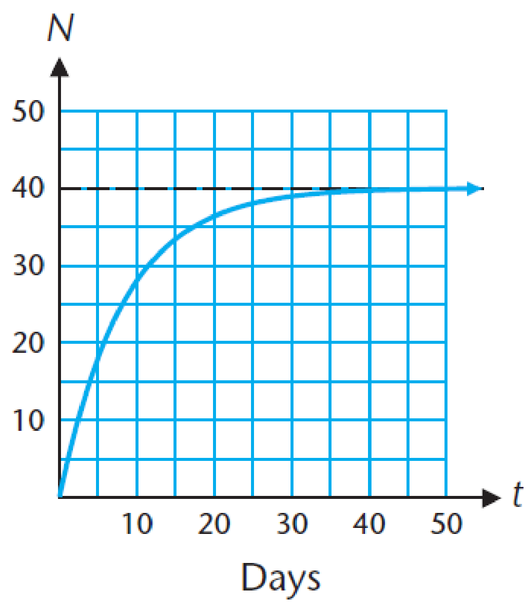
ONLINE COLLEGES

13. Learning Curve

People assigned to assemble circuit boards for a computer manufacturing company undergo on-the-job training **.** From past experience , it was found that the learning curve for the average employee is given by

$$N(t) = 40 \cdot (1 - e^{-0.12t})$$

where ***N*** is the number of boards assembled per day after ***t*** days of training. (Figure below!)



a) How many boards can an average employee produce after 3 days of training?

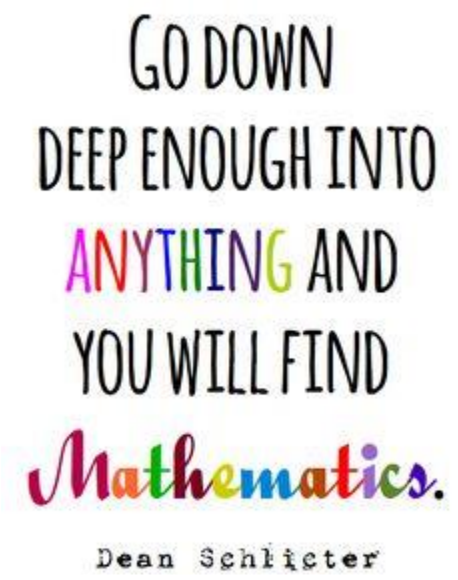
____________________________ *(Rounded to the nearest integer.)*

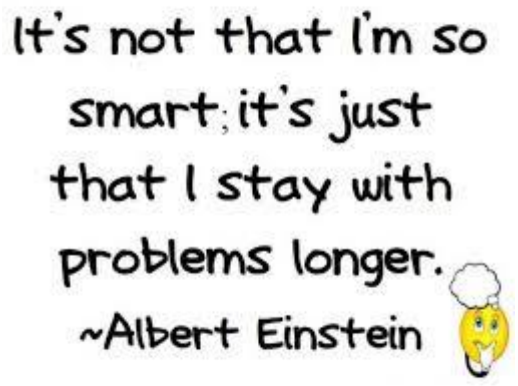
b) How many boards can an average employee produce after 5 days of training?

____________________________ *(Rounded to the nearest integer.)*

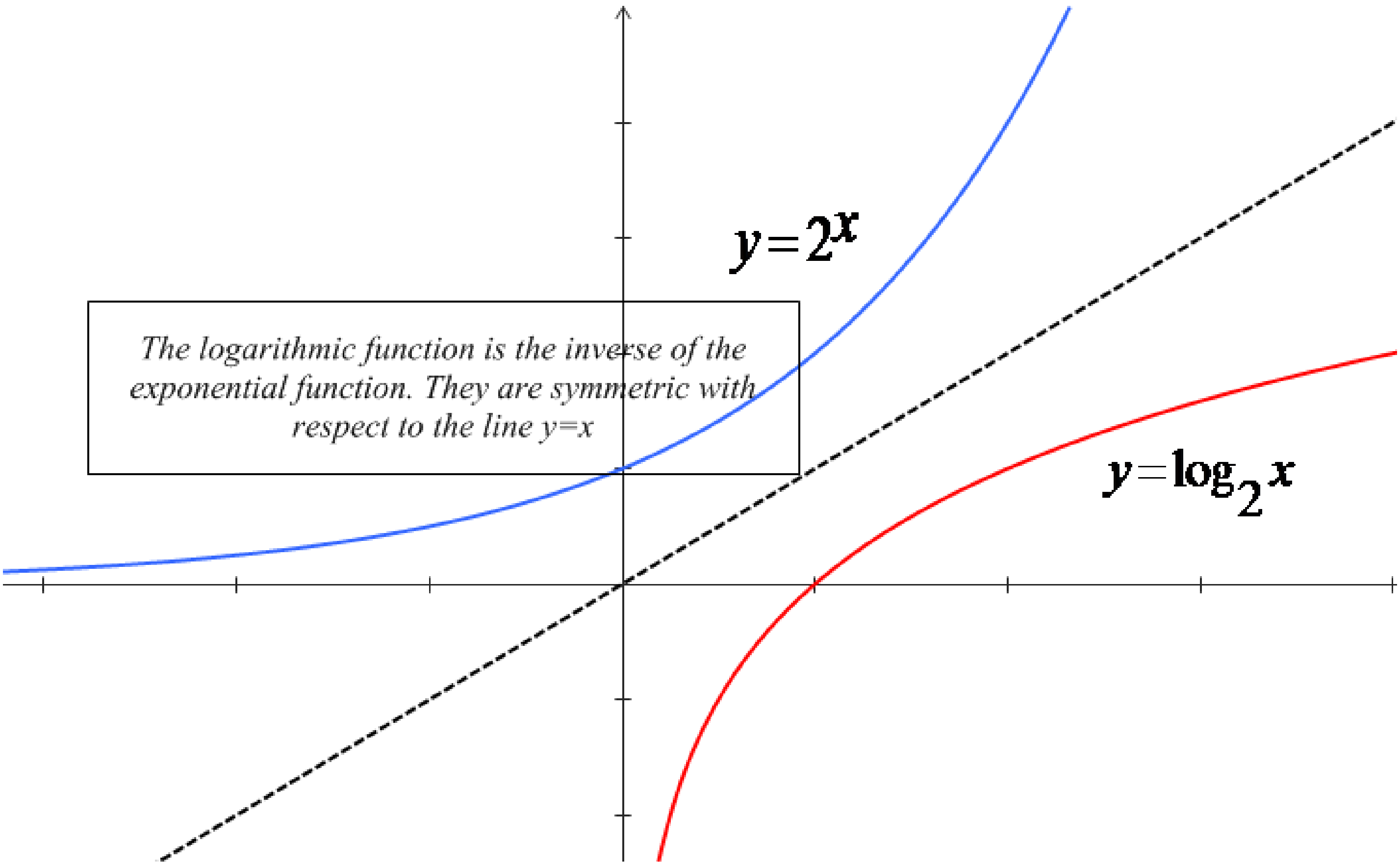
c) Which value is the limiting one if ***t*** increases without bound?

____________________________ boards per day

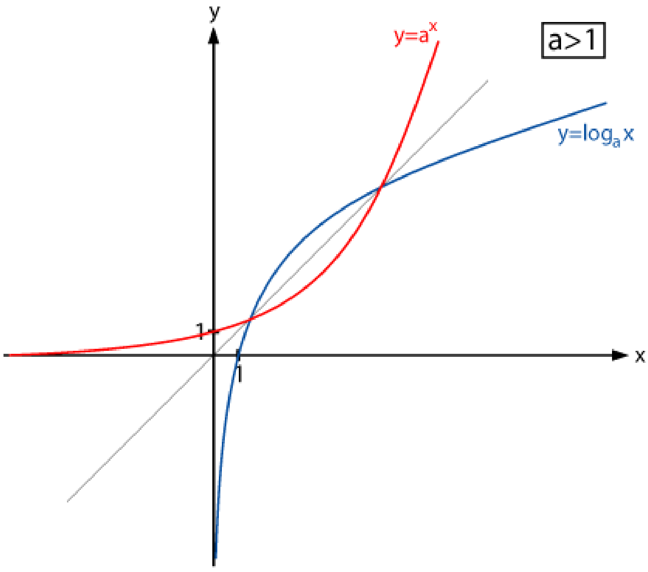


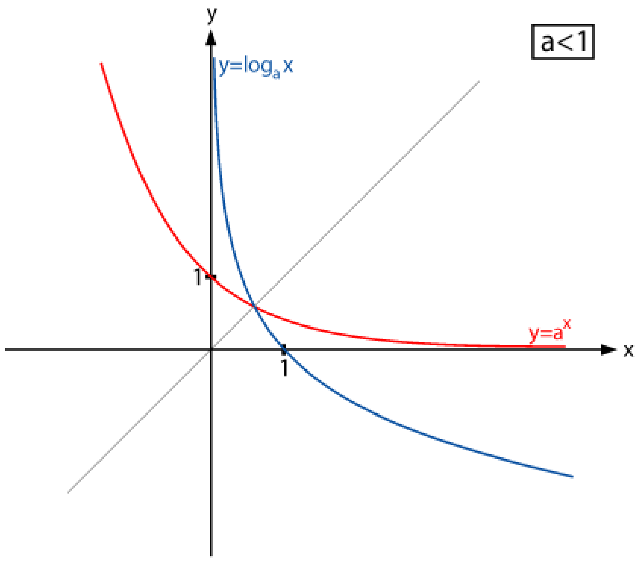


Logarithmic Functions

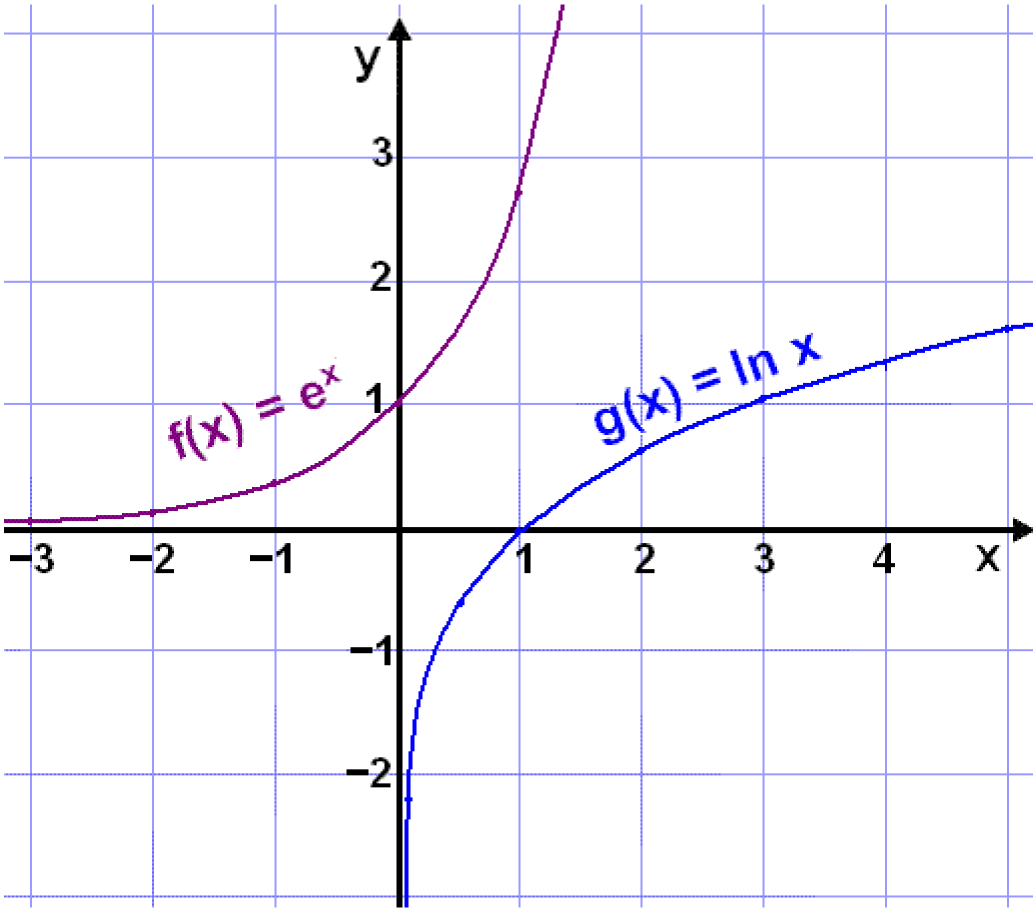


Exponential Laws	**Logarithm Laws**
$x^a \cdot x^b = x^{a+b}$	$\log(ab) = \log(a) + \log(b)$
$\dfrac{x^a}{x^b} = x^{a-b}$	$\log\left(\dfrac{a}{b}\right) = \log(a) - \log(b)$
$(x^a)^b = x^{ab}$	$\log(a^b) = b \cdot \log(a)$
$x^{-a} = \dfrac{1}{x^a}$	$\log_x\left(\dfrac{1}{x^a}\right) = -a$
$x^0 = 1$	$\log_x 1 = 0$





miStAkEs
are proof
that you are
TRYING



14. Which of the following statements is not correct?

- ◯ $\log_{10} 10 = 1$
- ◯ $\log (2 + 3) = \log (2 \cdot 3)$
- ◯ $\log_{10} 1 = 0$
- ◯ $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

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15. If $\log \frac{a}{b} + \log \frac{b}{a} = \log (a + b)$, then:

- ◯ $a + b = 1$
- ◯ $a - b = 1$
- ◯ $a = b$
- ◯ $a^2 - b^2 = 1$

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16. If $\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$, then x is equal to:

- ◯ $-\frac{3}{4}$
- ◯ $\frac{3}{4}$
- ◯ $\frac{256}{81}$
- ◯ $\frac{81}{256}$

17. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of ***y*** is:

◯ 2^{10} ◯ 2^{100} ◯ 2^{1000} ◯ 2^{10000}

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18. If $a^x = b^y$, then:

◯ $\log\frac{a}{b} = \frac{x}{y}$ ◯ $\log\frac{a}{b} = \frac{y}{x}$ ◯ $\frac{\log a}{\log b} = \frac{y}{x}$ ◯ $\frac{\log a}{\log b} = \frac{x}{y}$

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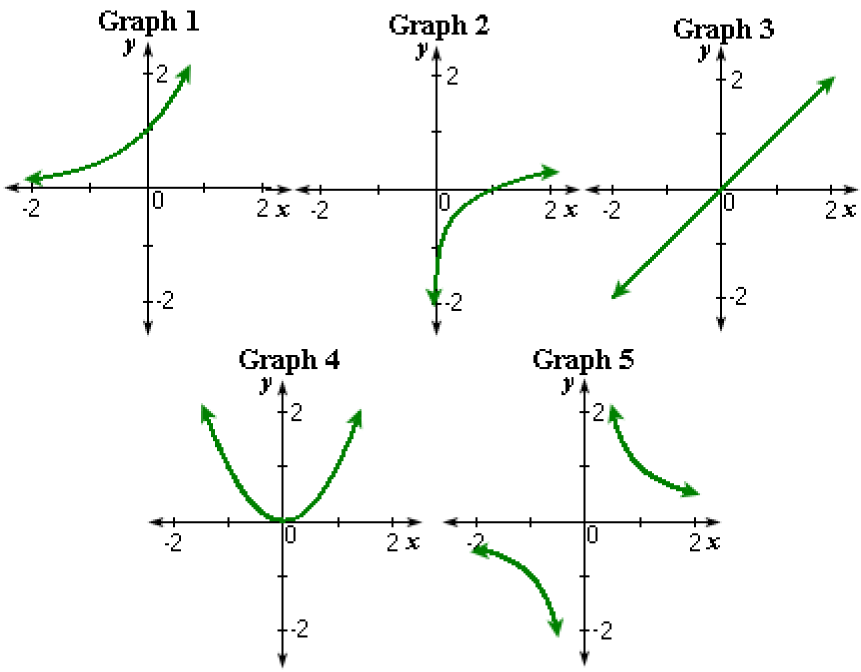
19. Identify the basic exponential and logarithmic function from the graphs below.



Exponential function is represented by the *Graph* ______.

Logarithmic function is represented by the *Graph* ______.

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20. The equation $3^x = 4$ has a solution:

◯ $x = \frac{4}{3}$ ◯ $x = \log_3 4$ ◯ $x = \log_4 3$ ◯ $x = \log\frac{4}{3}$

21. The domain of the function $f(x)=\log_x(6-x)$ is:

◯ $x<0$ ◯ $x<6$ ◯ $0<x<6, x\neq 1$ ◯ $1<x<6$

---

22. The graph of $f(x)=4^x$ and the graph of $g(x)=\log_4 x$ are symmetrical with respect to the line:

◯ $x=0$ ◯ $y=0$ ◯ $y=x$ ◯ $y=-x$

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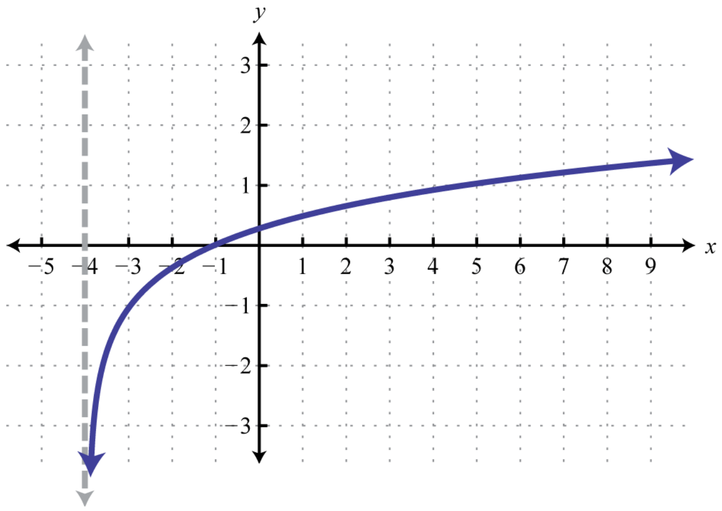
23. The graph of the logarithmic function shown below has:



◯ A vertical asymptote at $x=-4$ ◯ A vertical asymptote at $x=-1$

◯ A horizontal asymptote at $y=-4$ ◯ A horizontal asymptote at $y=-1$

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24. Find the range of $y=\log x+2$

◯ $[1,+\infty)$ ◯ $[2,+\infty)$ ◯ $(2,+\infty)$ ◯ All Reals

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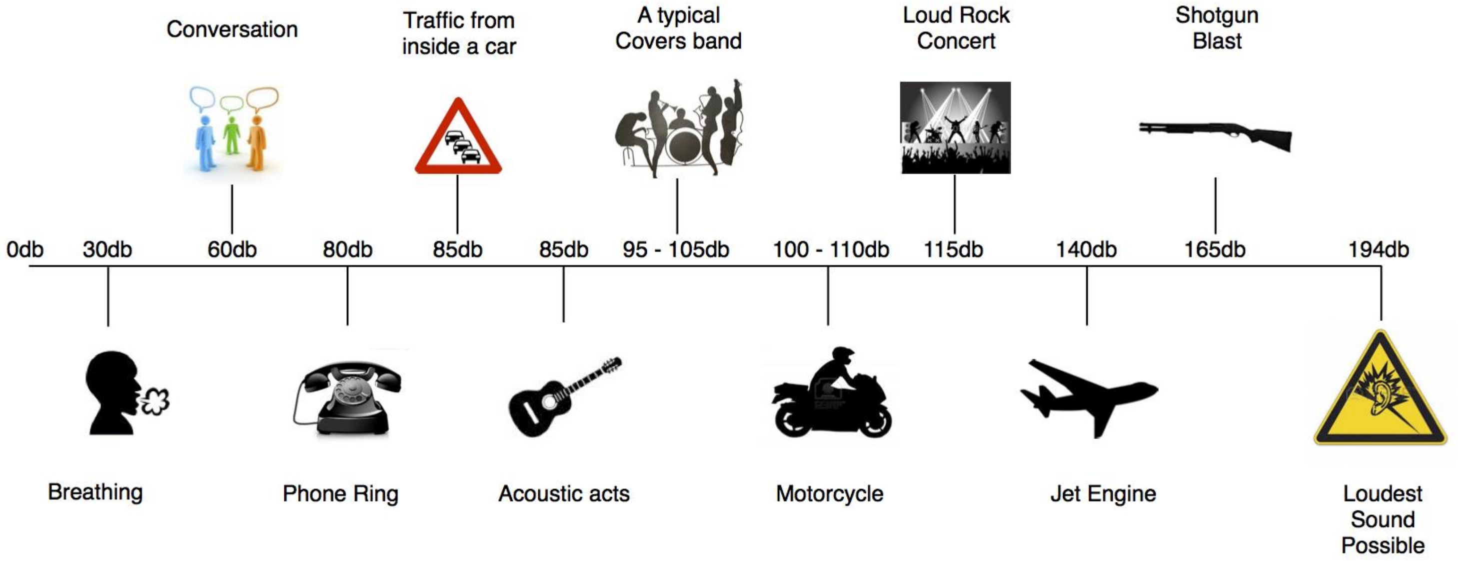
25. If $f(x)=e^{3\ln x}$, then $f(-2)$ is equal to:

◯ -8 ◯ -2 ◯ 8 ◯ does not exist



26. The decibel level of a sound may be calculated using the formula

$$L = 10 \cdot \log(10^{12} \cdot I)$$

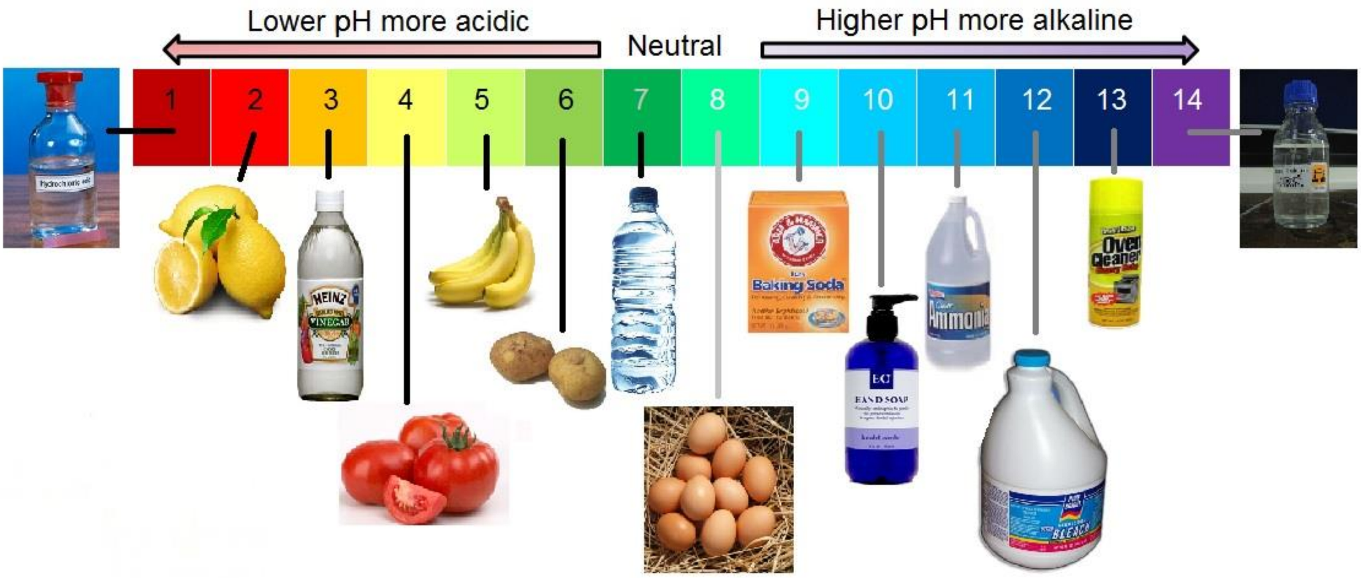
where ***L*** is the loudness of the sound *(dB)* and ***I*** is the intensity of the sound.

a) An equation that can be used to solve the value of ***I*** is:

◯ $I = \dfrac{L}{120 \cdot \log 10}$ ◯ $I = 10^{12} \cdot \log\left(\dfrac{L}{10}\right)$ ◯ $I = 10^{\frac{L-120}{10}}$ ◯ $I = \dfrac{L}{10^{13}}$

b) The loudness of the jet engine is 150 *dB*. The magnitude of the sound intensity is:

◯ 1.25 ◯ $1.18 \cdot 10^{12}$ ◯ 1000 ◯ $1.5 \cdot 10^{-11}$



The measure of acidity of a liquid is called the pH of the liquid. This is based on the amount of hydrogen ions **[H⁺]** in the liquid. The formula for pH is:

$$pH = -\log[\,H^+\,]$$

where **$[\,H^+\,]$** is the concentration of hydrogen ions, given in a unit called mol/L ("moles per liter"; one mole is $6.022 \cdot 10^{23}$ molecules or atoms). Liquids with a low pH (down to 0) are more acidic than those with a high pH. Water, which is neutral (neither acidic nor alkaline, the opposite of acidic) has a pH of 7.0.

27. **a)** Find the pH of milk, to the nearest tenth, whose concentration of hydrogen ions,

$[\,H^+\,] = 4 \cdot 10^{-7}$ mol/L.

pH ≈ _______________.

b) If lime juice has a pH of 1.7, what is the concentration of hydrogen ions (in mol/L) in lime juice, to the nearest hundredth?

The concentration of hydrogen ions in lime juice is________.

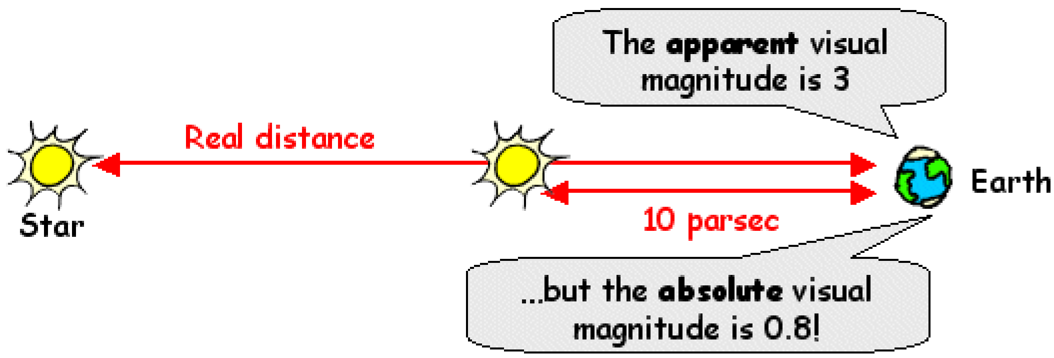
ABSOLUTE MAGNITUDE

The **absolute magnitude** of a **star**, ***M*** is the magnitude the star would have if it was placed at a **distance** of 10 parsecs from Earth. By considering **stars** at a fixed distance, **astronomers** can compare the real (intrinsic) brightnesses of different stars. The term absolute magnitude usually refers to the absolute visual magnitude, M_v of the star, even though the term 'visual' really restricts the measurement of the brightness to the **wavelength** range between 4000 and 7000 Angstroms.

To convert the observed brightness of a star (the **apparent magnitude**, ***m***) to an absolute magnitude, we need to know the distance, ***d***, to the star. Alternatively, if we know the distance and the apparent magnitude of a star, we can calculate its absolute magnitude. Both calculations are made using the formula

$$m - M = 5\log\frac{d}{10}$$

Unit	Abbreviation	Conversion
Astronomical Unit	AU	1 AU = 1.5×10^{11} m
Light Year	lyr	1 ly = 9.46×10^{15} m
Parsec	pc	1pc = 3.08×10^{16} m
		1 pc = 3.26 ly or 1 pc = 206265 AU



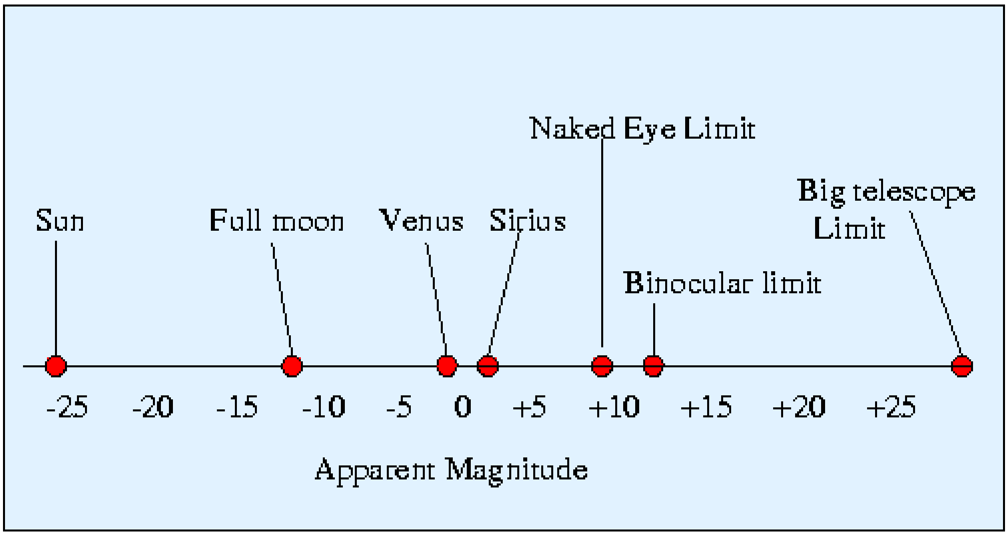
28. a) A star is 370 light years away. What is the distance in parsec?

◯ 113 ◯ 370 ◯ 1070 ◯ 3700

b) A star has an apparent magnitude of -1.6, and is known to be 200 light years away.

What is its absolute magnitude?

◯ -1.30 ◯ -5.54 ◯ -6.14 ◯ -8.10



c) Bellatrix and Elinath are two stars with the same apparent magnitude. The distance from Earth to Bellatrix is 470 light years and its absolute magnitude is -4.2.

(i) Calculate the distance to Bellatrix in parsecs.
(Rounded to the nearest integer.)

________________________ parsecs

(ii) Calculate , to the nearest tenth, the apparent magnitude of Bellatrix.

____

(iii) Elinath has an absolute magnitude of -3.2. Which of these two stars is closer to Earth?

◯ Bellatrix ◯ Elinath

At the end...

Mark the emotion that shows how you feel.

This test was: